

Exchange Rate Systems and Cross Currency Pricing in 2025 and 2026**Daily euro reference rates, triangular conversion, and trade adjustment**Author: Sami Ghanem¹*Affiliation:*

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Abstract:

This paper compares the euro exchange rates of the Danish krone, US dollar, and pound sterling over 438 European Central Bank (ECB) observation dates from 2 January 2025 to 18 September 2026. It combines descriptive statistics with a cross-currency pricing exercise and a discussion of trade adjustment. The standard deviation of daily log changes was 0.98 basis points for EUR/DKK, 44.23 for EUR/USD, and 25.93 for EUR/GBP. One euro gained 0.23% in kroner, 11.04% in dollars, and 3.32% in pounds over the sample. The implied dollar price of one pound increased from 1.2417 to 1.3344. These results are consistent with a stable krone against the euro, but they do not isolate the causal effect of exchange-rate policy. A triangular conversion built entirely from the same reference rates returns the starting amount by construction and cannot demonstrate an executable arbitrage profit. The Marshall–Lerner condition provides a framework for discussing trade adjustment, although trade elasticities are not estimated. The analysis shows how reference rates support currency comparisons and valuation while identifying the additional evidence needed for trading and trade-balance conclusions.

Keywords:

Exchange-rate systems, Euro, Danish krone, Cross-currency pricing, Triangular arbitrage, Marshall–Lerner condition

Introduction:

An exchange-rate movement changes what a foreign-currency payment is worth before a business changes the price or quantity of anything it sells. A euro-area exporter receiving dollars, for example, must convert those receipts into euros to cover domestic expenses. A Danish company paying a euro invoice faces a different pattern of currency movements because the krone is kept close to the euro. These examples make exchange-rate systems relevant to routine financial decisions, including budgeting, borrowing, and comparing international prices.

This paper examines the Danish krone, US dollar, and pound sterling using a common euro reference. Denmark's policy is to maintain a very stable krone–euro exchange rate, whereas the dollar and pound have no comparable commitment to the euro (Danmarks Nationalbank, n.d.-a). The comparison therefore brings together a currency linked to the euro and two currencies whose euro prices can move more freely. The sample covers 2 January 2025 to 18 September 2026, allowing the same dates and measurement rules to be used for all three currencies.

The research problem is that an exchange-rate table can support several different types of calculation, but those calculations do not all answer the same economic question. A change in a currency's price is directly observable. The reason for that change is harder to identify. Similarly, dividing two euro rates produces a cross rate, but does not establish that a trader could buy and sell at those prices. A further step is needed before linking a currency movement to exports, imports, or the trade balance.

I address three questions. How do the currencies compare in daily variability and beginning-to-end movements? What does the implied dollar–pound rate show about cross-currency pricing and triangular conversion? Finally, what can the Marshall–Lerner framework explain about trade adjustment, and which of its inputs are absent from this dataset? The objectives are to report transparent descriptive measures, show the conversion arithmetic, and connect the findings to international-finance theory without treating the theory as an estimated result.

The study uses 438 common European Central Bank (ECB) observation dates and 437 consecutive observed-day changes. Its contribution is a focused application of course concepts to one consistent sample, rather than a forecast or a causal comparison of exchange-rate regimes. The next section reviews the institutional and academic background. The methodology defines the quotations and calculations, followed by the results and their practical interpretation. Separate discussion and conclusion sections then assess the findings, limitations, and scope for further research.

Literature Review:

Exchange rate systems and the currency of measurement

Exchange-rate systems differ in the extent to which the authorities commit to maintaining a particular external currency value. A fixed arrangement directs policy toward an exchange-rate objective; a floating arrangement allows the price to adjust without that specific commitment. These categories help organize a comparison, although an observed period of stability does not by itself identify a formal regime. Institutional evidence is needed alongside the price series.

Denmark provides that institutional context. Danmarks Nationalbank (n.d.-a) identifies a central rate of DKK 7.46038 per euro and explains that the krone can fluctuate against currencies other than the euro. Stability therefore has a specific reference currency. A Danish firm with euro expenses and a Danish firm with dollar expenses do not necessarily face the same exchange-rate exposure. This distinction matters when interpreting the relatively flat DKK series in the present study.

The policy also involves operational choices. Danmarks Nationalbank (n.d.-b) explains that it can buy or sell currency and adjust policy interest rates to influence the krone's euro value. The institutional commitment offers a plausible explanation for low EUR/DKK variability. However, its causal contribution cannot be separated from other influences simply by comparing Denmark with the United States and United Kingdom over one period. These economies differ in more than their exchange-rate arrangements.

Reference rates and cross currency consistency

The nature of the quotation is as important as the exchange-rate system. The ECB (2026a) publishes reference rates for information and discourages their use for transactions. An official daily rate is suitable for documenting movements on a consistent basis. A transaction price, by contrast, must be available to a particular buyer or seller for an amount that can actually be exchanged. The distinction prevents reference-rate arithmetic from being interpreted as a trading result.

Cross-currency pricing follows from matching units. If one euro buys a stated number of dollars and pounds, dividing dollars per euro by pounds per euro gives dollars per pound. The euro cancels from the ratio. This relationship connects the three quotations and supplies a benchmark for checking conversion calculations. It also explains why a triangle constructed from only two independent prices necessarily balances. A genuine test of market consistency needs a third, independently observed price; otherwise the result has been imposed by the calculation itself.

Triangular arbitrage and execution

Fenn et al. (2009) examine triangular arbitrage using high-frequency executable foreign-exchange prices. They find opportunities that are generally small and short-lived, with profitability sensitive to execution. Their study is relevant because its evidence comes from prices at which transactions could potentially occur, rather than a daily reference table. It supports a careful distinction between identifying a mathematical price discrepancy and capturing a net trading profit.

A trader also faces a bid–ask spread. The dealer's bid is the price received when selling the base currency; the ask is the price paid when buying it. Using the favorable side on every leg would

overstate the amount available at the end of a conversion. Fees, trading limits, and price changes while the orders are being filled can further reduce an apparent gain. These practical requirements explain why the present paper treats its constructed triangle as an illustration of pricing consistency.

AI and its impact on other aspects of finance

Beainy and Kamel (2026) argue that in the modern system, Cryptocurrency and Gold are increasingly replacing currencies as reserves in foreign banks as well as for individuals (Aoun et al., 2026). In the Eurozone, the value of the euro, interest rates and the cost of capital heavily impact firm performance, especially SMEs (Rizk & Sassine, 2023), investor sentiments may also affect currency and pricing (Khalife et al., 2025).

Kamel et al. (2026) discuss how digitalization impact not only sustainability, firm performance, but the economy at a macro level. Mawad and Freiha (2024) argument about the importance of a stable financial system, including currency stability and banking services for firms performance.

Finally, while the devaluation of some currencies have a limited impact, some currencies such as the United States Dollar have a significant effect on the entire financial system (Kamel et al., 2025).

Trade adjustment and the Marshall Lerner condition

The traditional Marshall–Lerner condition states that a depreciation improves the trade balance when the absolute export and import demand elasticities sum to more than one, under assumptions including initially balanced trade, full exchange-rate pass-through, and sufficiently elastic supply. Its logic is that export and import quantities must respond enough to offset the higher domestic-currency cost of imports. The International Monetary Fund (IMF, 2007) shows why incomplete pass-through complicates this benchmark. The condition is therefore a conditional result, rather than a rule that every depreciation improves trade.

Timing also matters. Existing contracts and slow quantity responses can produce an initial deterioration before later improvement, commonly described as the J-curve (IMF, 2007). A period of depreciation alone is insufficient to identify that pattern. Trade values and volumes must be observed over time, and changes in demand must also be considered.

Leigh et al. (2017) find that the relationship between exchange rates and trade remains economically relevant, while financial conditions and economic slack affect the response. Li and Meleshchuk (2024) extend the discussion to services and show that both bilateral and dollar exchange rates matter. Their evidence on dominant currency pricing makes the invoice currency relevant even when the trading partners are outside the United States.

Together, these studies explain why a nominal bilateral rate is only one input into a trade analysis. The present dataset can document relative currency prices and construct a cross rate. It cannot supply independent transaction quotes or estimate trade-demand elasticities. Keeping these questions separate allows the empirical section to remain focused while the discussion draws on the wider literature.

Methodology:

Data selection and quotation convention

The dataset is the ECB historical euro reference-rate CSV downloaded on 20 September 2026 (ECB, 2026b). The sample retains dates from 2 January 2025 through 18 September 2026 with USD, GBP, and DKK values available together. There are 438 complete observation dates. Sorting them in ascending order produces 437 consecutive observed-day changes. The end date is held fixed throughout the analysis, including the table, chart, and cross-rate calculations.

Each rate is expressed as units of foreign currency per euro. A rise in USD per EUR therefore means that the euro buys more dollars. It is a euro appreciation against the dollar and, equivalently, a dollar depreciation against the euro. The percentage changes of a quotation and its reciprocal have different magnitudes, so the reported percentages always retain their stated direction. To avoid ambiguity, the tables write the units explicitly.

The analysis follows publication dates without inserting weekend or holiday observations. A consecutive observed-day change can therefore span more than one calendar day. In this paper, “daily” refers to these successive reference-rate observations, rather than a complete sequence of calendar days or intraday trading returns. No missing calendar dates are filled with repeated or interpolated prices.

Measures of change and variability

Let $R(c,t)$ denote the number of units of currency c per euro on observation date t . Daily log changes are calculated from adjacent observations:

$$r(c,t) = \ln[R(c,t) / R(c,t-1)]$$

The full-sample variability measure is the sample standard deviation of these 437 changes, using the sample denominator of 436, multiplied by 10,000 to express it in basis points. One basis point equals 0.01 percentage point. This transformation compares proportional variability rather than the standard deviation of rate levels, which have different units.

$$\text{Daily SD in basis points} = 10,000 \times \text{SD}[r(c,t)]$$

Beginning-to-end change is calculated as follows:

$$\text{Percentage change} = 100 \times [R(c,\text{last}) / R(c,\text{first}) - 1]$$

Minimum and maximum observed levels complement these measures. The standard deviation describes dispersion of changes, while the endpoint percentage describes the net movement over the whole sample. Neither measure is annualized or presented as a forecast.

Cross rates and the conversion framework

The implied dollar price of one pound is calculated for every common date using the two euro rates. The label “USD per GBP” is used consistently, since a slash notation can otherwise be confused with a market pair name. The calculation is:

$$C(t) = R(\text{USD}, t) / R(\text{GBP}, t)$$

Beginning-to-end change, observed range, and daily log-change standard deviation are calculated for this cross rate in the same way as for the euro rates. Because its log change equals the USD-per-EUR log change minus the GBP-per-EUR log change, its variability depends on how the two component changes move together. It should not be obtained by adding or subtracting their standard deviations.

The triangular exercise begins with one euro, converts into dollars, then pounds, and finally euros. Each conversion factor is derived from the same-date reference rates:

$$1 \times R(\text{USD}) \times [R(\text{GBP}) / R(\text{USD})] \times [1 / R(\text{GBP})] = 1$$

All intermediate factors cancel. Any tiny discrepancy caused by rounding displayed numbers is numerical, not evidence of an available profit. An executable test would replace the constructed cross rate with an independent quote and use synchronized bid and ask prices on the appropriate sides, together with fees and information about available trading quantities.

Chart construction and analytical limits

Figure 1 indexes each euro rate to 100 on 2 January 2025, using 100 times the current rate divided by its initial rate. Indexing puts proportional movements on a common scale without changing the underlying observations. Separate 2025 and 2026 summaries provide a descriptive comparison across subperiods; 2026 ends on 18 September and is not a full year.

The Marshall–Lerner benchmark is $|\eta_X| + |\eta_M| > 1$, where η_X and η_M represent export and import demand price elasticities. These parameters are discussed, not estimated. The dataset contains no trade prices, quantities, invoice-currency information, or trade balance. It also contains no policy controls or counterfactual comparison that would identify the effect of a currency regime. Accordingly, the method reports associations and arithmetic relationships. A causal regime study, a trading test, and a trade-elasticity study would each require additional observations suited to its own question.

Results:

Full sample currency movements

Table 1

Full-sample summary of ECB reference rates, 2 January 2025 to 18 September 2026

Rate	Start	End	Change	Daily SD, bp	Observed range
USD per EUR	1.0321	1.1460	+11.04%	44.23	1.0198–1.1974
GBP per EUR	0.83118	0.85880	+3.32%	25.93	0.8253–0.8846
DKK per EUR	7.4583	7.4754	+0.23%	0.98	7.4581–7.4759
USD per GBP, implied	1.24173	1.33442	+7.46%	44.98	1.21174–1.38167

Note. N = 438 rates and 437 daily changes for each row. Daily SD is the sample standard deviation of log changes multiplied by 10,000. Cross rates use the same-date euro rates. Source: ECB (2026b); author calculations.

Table 1 shows a clear ordering of daily variability. USD per EUR has a standard deviation of 44.23 basis points, followed by GBP per EUR at 25.93 and DKK per EUR at 0.98. The dollar and pound rates therefore varied considerably more than the krone rate over the common sample. These are comparisons of movements against the euro; they do not rank every form of financial risk associated with the currencies.

The endpoint changes show that one euro bought 11.04% more dollars, 3.32% more pounds, and 0.23% more kroner at the end than at the beginning. The dollar rate increased from 1.0321 to 1.1460, while the pound rate increased from 0.83118 to 0.85880. These changes describe euro appreciation against both currencies. They should not be reported as investment returns without considering the investor's currency, interest income, and transaction costs.

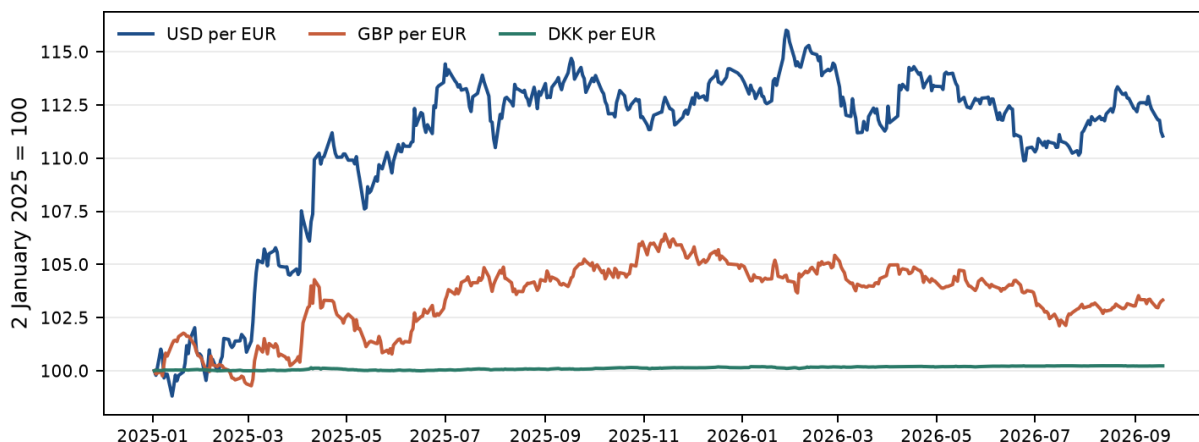
The observed ranges add information that endpoints omit. USD per EUR moved between 1.0198 and 1.1974, so its final value was below the sample maximum despite its positive net change. GBP per EUR also ended below its maximum of 0.8846. DKK per EUR stayed within 7.4581–7.4759, a much narrower proportional interval. A business exposed during the sample could consequently have experienced conversions quite different from either endpoint.

The implied dollar price of one pound rose by 7.46%, from 1.24173 to 1.33442. Sterling strengthened against the dollar even though it weakened against the euro. There is no contradiction: a currency's performance depends on the comparison currency. The cross rate's daily standard deviation of 44.98 basis points also shows why exposure between dollars and pounds deserves its own calculation.

The exchange rate path and reproducibility checks

Figure 1

Euro reference rates indexed to 100 on 2 January 2025



Note. Each series is $100 \times$ its rate divided by its rate on 2 January 2025. Source: ECB (2026b); author calculations.

Figure 1 shows repeated rises and reversals in the dollar and pound series, while the krone remains close to its starting index. The year-specific daily standard deviations for USD, GBP, and DKK are 49.38, 29.74, and 1.10 basis points in 2025, and 35.74, 19.45, and 0.79 in 2026 through 18 September. The respective years contain 255 and 183 rate observations. The same variability ordering appears in both subperiods, although their unequal coverage limits comparison.

Table 2

Selected reproducibility checkpoints from the daily archive

Date	USD per EUR	GBP per EUR	DKK per EUR	USD per GBP
2 Jan 2025	1.0321	0.83118	7.4583	1.24173
31 Dec 2025	1.1750	0.87260	7.4689	1.34655
31 Mar 2026	1.1498	0.86833	7.4730	1.32415
18 Sep 2026	1.1460	0.85880	7.4754	1.33442

Note. Source: ECB (2026b); author cross-rate calculations.

On 18 September 2026, 1.1460 divided by 0.85880 gives 1.33442 USD per GBP. The constructed cycle is $\text{€}1 \rightarrow \$1.1460 \rightarrow \text{£}0.85880 \rightarrow \text{€}1.0000$ before costs and subject to rounding. The return to one euro is an algebraic identity, not a measured arbitrage profit.

Discussion and Conclusion:

The low variability of DKK per EUR is consistent with Denmark's stated exchange-rate policy. That agreement between the institutional description and the data is useful, but it does not quantify what the krone would have done under a different policy. The comparison contains one euro-linked currency and two different economies with more flexible euro rates. Interest-rate expectations, changes in demand, and other influences could affect their observed paths. A claim that the regime alone caused the variability difference would therefore go beyond this research design.

For a firm, the results become meaningful when connected to a specific cash flow. Consider an illustrative euro-area exporter with a fixed \$10,000 receivable. Conversion at the opening reference rate would produce approximately €9,688.98; conversion at the closing rate would produce €8,726.00. The €962.98 difference follows directly from dividing the same dollar amount by the two USD-per-EUR rates. It illustrates valuation exposure, rather than an observed company loss. An importer making a fixed dollar payment would experience the opposite benefit from that euro appreciation, before fees or hedging.

The krone comparison suggests much less variation for an equivalent euro–krone conversion during this window. It does not mean that every Danish business had little currency exposure: dollar invoices, foreign-currency debt, or mismatched receipts and payments can still matter. Firms would need to identify the currency and timing of their net cash flows before deciding how much exposure to hedge. Historical daily variability is one input to that decision, not a complete measure of future losses.

The cross-rate exercise has a similarly practical but limited use. It supplies a benchmark for checking whether a conversion has the right units and direction. It cannot establish that an available dealer quote was mispriced. The constructed triangle contains no independent third price and no bid–ask spreads. Its exact balance consequently provides no empirical test of market efficiency, even though it correctly illustrates the no-arbitrage pricing relationship.

Trade adjustment requires another set of evidence. The euro's appreciation against the dollar does not prove that euro-area exports fell or that US exports rose. Invoice currencies, pass-through, demand, and supply responses intervene between exchange rates and trade outcomes (Leigh et al., 2017; Li & Meleshchuk, 2024). A useful extension would combine a longer series of trade prices and volumes with real effective exchange rates and lagged demand controls. A separate extension could collect synchronized executable currency quotes. These approaches would test questions that daily reference rates can only motivate.

This paper examined exchange-rate systems and cross-currency pricing using 438 common ECB observation dates from January 2025 to September 2026. The first research question concerned differences in euro-rate movements. DKK per EUR was far less variable than USD per EUR or GBP per EUR: the respective daily standard deviations were 0.98, 44.23, and 25.93 basis points. The same ordering appeared in the separate year summaries. This evidence describes a stable krone–euro relationship over the sample and is consistent with Denmark's institutional commitment.

The beginning-to-end results answer a different part of that question. One euro gained 0.23% in kroner, 11.04% in dollars, and 3.32% in pounds. The retained chart shows why these endpoint changes should be read alongside the intervening path. A positive net movement can include substantial reversals, and the final observation need not represent the largest exposure experienced

during the period. Businesses evaluating currency risk would therefore need both the cash-flow horizon and the relevant quotation.

The second question concerned cross-currency pricing. The implied USD-per-GBP rate increased from approximately 1.2417 to 1.3344, showing sterling appreciation against the dollar despite sterling depreciation against the euro. This result reinforces the importance of stating the base and quote currencies clearly. The triangular conversion returned exactly the initial euro because all legs were derived from the same two reference rates. The exercise establishes the arithmetic relationship; an arbitrage conclusion would require independently observed executable prices and trading costs.

The third question concerned trade adjustment. The Marshall–Lerner condition helps explain why changes in relative prices must be considered alongside export and import responses. The paper does not estimate those elasticities or identify a J-curve. It consequently makes no claim that the observed currency movements improved or worsened a country's trade balance. The distinction preserves the usefulness of the theory without presenting an untested mechanism as a finding.

The main contribution is a transparent connection between an official rate dataset and several FIN420 concepts. Its limits are equally specific: a relatively short sample, informational prices, and no trade-flow or causal identification model. Further work should address each missing input directly rather than draw stronger conclusions from the same rates. For financial decisions, the study supports careful quotation, exposure measurement, and conversion checks. For research, it shows where descriptive evidence ends and where a different dataset and method are necessary.

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